Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year: 2017-2018 Date: 6 / 1 / 2018	Minoufia University	Subject : Numerical Analysis Code: BES 512 Time Allowed: 3 hours Year : Master Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 [25]

(A) State the Classification of Partial Differential Equations? And state the various types of boundary conditions?

(B) Write brief notes on the following topics:

i) Consistency.

ii) Stability.

iii) Convergence.

iv) Lax's equivalence theorem.

(C) The governing equations of motion for one-dimensional, inviscid flows are given by the Euler equations. If the assumption of perfect gas is imposed, the system is written as:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho\, a^2\frac{\partial u}{\partial x} = 0$$

Classified this system?

Q.2 [25]

(A) Determine the approximate forward difference representation for $\partial^3 f/\partial x^3$ which is of the order (Δx), given evenly spaced grid points f_i, f_{i+1}, f_{i+2}, f_{i+3} by means of:

i) Taylor series expansion.

ii) Forward difference recurrence formula.

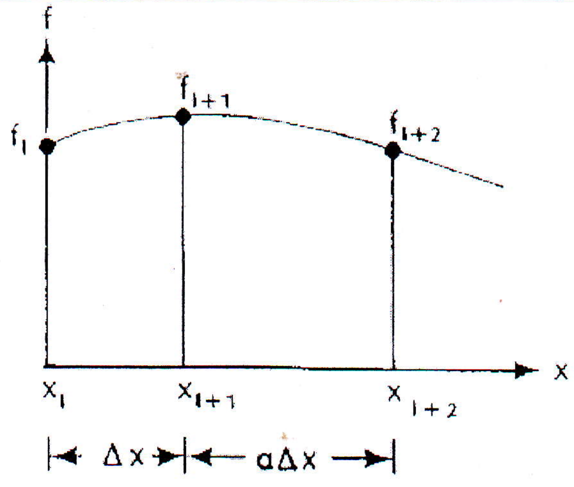
iii) A third-degree polynomial passing through the four points.

(B) For the function $f(x) = sin(2\pi x)$, determine $\partial f/\partial x$ at $x = 0.375$ using central difference representation of order $(\Delta x)^2$ and order $(\Delta x)^4$. Use step sizes of 0.01, 0.1 and 0.25. Compare the result with the exact analytical solution and discuss the results.

(C) Derive a second-order forward difference approximation for $\partial f/\partial x$ with unequally spaced grid points, as shown in figure.

i) Using Taylor series Expansion.

ii) A second-degree polynomial passing through the points.



Unequally spaced grid points

Q.3 **(A)** Derive a first-order central difference approximation for the mixed partial derivatives $\partial 2f/\partial x\partial y$. [25]

i) Using Taylor series Expansion.

ii) Use of partial derivatives with respect to one independent variable.

(B) Using 4th order Runge-Kutta method to find $x(0.2), y(0.2)$

$$\frac{dx}{dt} = x^2 + y + 5t, \qquad x(0) = 2.0$$

$$\frac{dy}{dt} = x + y^2 - 6t, \qquad y(0) = 3.0$$

(C) Given the modal of wave equation:

$$\frac{\partial u}{\partial t} = -\alpha \frac{\partial u}{\partial x}$$

Using central differencing and apply the Von Neumann stability analysis to illustrate the application of stability analysis to the three-level FDEs

Q.4 **(A)** The governing equation of a uniform Bernoulli–Euler beam under pure bending resting on fluid layer under axial force is: [25]

$$\frac{\partial^4 w}{\partial x^4} + P\frac{\partial^2 w}{\partial x^2} + K_f w + F(x,t) = 0, \quad 0 \le x \le L.$$

with boundary conditions (Clamped–Simply supported):

$$at \;\; x = 0, \qquad W(x) = 0$$

$$at \;\; x = 0, \qquad \frac{dW(x)}{dx} = 0$$

$$at \;\; x = L \qquad W(x) = 0$$

$$at \;\; x = L \qquad \frac{d^2W(x)}{dx^2} = 0$$

Solve the beam equation problem using the adomian decomposition method (ADM). Then compared the results with exact solutions, in the following form:

$$F(x,y) = 1.$$

(B) define and gives examples of:

i) Discrete Perturbation Stability Analysis.

ii) Von Neumann Stability Analysis.

iii) Artificial Viscosity.

(C) State the application and limitations of the von Neumann stability analysis

This exam measures the following ILOs								
Question Number	Q1-a	Q1-b	Q3-b	Q4-a	Q1-c	Q2-a	Q3-a	Q4-c
	Q4-b				Q2-b	Q2-c	Q3-c	
	Knowledge &understanding skills				Intellectual Skills		Professional Skills	

Good Luck

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